The Physics of Bowling:

How good bowlers stay off the straight and narrow.

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History:

- Forms of bowling can be traced back to Egypt around 3200 B.C., although the first "standardized rules" were established in New York in 1895.
- The origin of ten pin bowling is attributed to an 1841 law in Connecticut banning ninepin bowling, which was the form of bowling brought to America from Europe. (The extra pin prevented violation of the law.)
- Lane dimensions: Length: 60 feet, Width: 3.5 feet.
- Ball specifications: Circumference: 2.25 feet, Weight: up to 16 pounds.

This and other information can be found at:

`http://en.wikipedia.org/wiki/Bowling`

Scoring a game:

- There are ten *frames* in which up to two balls may be rolled toward the pins;
- The base score for a frame is the total *pinfall* (number of pins knocked down);
- In the case of a *strike* (all 10 pins knocked down on first ball) the pinfalls of the next two balls are added to the score for the frame;
- In the case of a *spare* (all remaining pins knocked down on second ball) the pinfall of the next ball is added to the score for the frame;
- These bonus pinfalls can lead to up to two additional frames when a spare or strike occurs in the tenth frame.

Strikes are king:

- A perfect game consists of twelve consecutive strikes, yielding a score of 30 for each frame and a total score of 300;
- The highest possible score without any strikes results from ten frames of 9-1 spares followed by an additional 9 in the eleventh frame, leading to a score of 19 for each frame and a total score of 190;
- In particular, consecutive strikes are essential to high scores:

1	2	3	4	5	6	7	8	9	10	Score
9/1	10/-	10/-	10/-	10/-	9/1	9/1	9/1	9/1	9/1/9	224
9/1	10/-	9/1	10/-	9/1	10/-	9/1	10/-	9/1	9/1/9	198

  In these two examples the frames are identical up to the ordering and there are four strikes in each game.

How to get a strike: (1 of 2)

- Experiment has shown that a angle of six degrees with respect to the lane boards is optimal for the generation of strikes (Wikipedia);
- The shot should make contact with the *pocket* (space between two front pins on either side);
- Assuming that the pocket is in the middle of the lane, the largest angle possible with a straight ball is

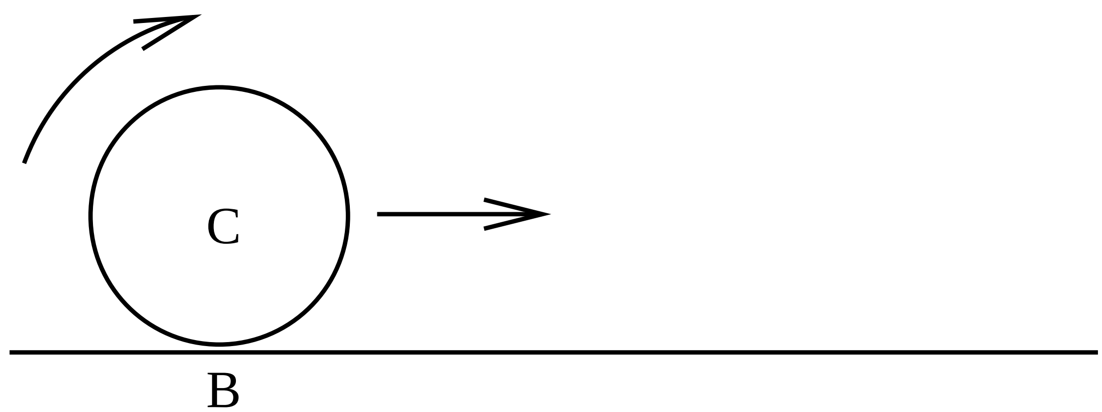
$$\tan^{-1}\left(\frac{1.75}{60}\right) \approx 1.67 \text{ degrees.}$$

How to get a strike: (2 of 2)

- A six degree angle with a straight shot would require the bowler to stand about 6.3 feet to the side of the pocket, which corresponds to a spot more than halfway across the adjacent lane. (D'oh!)
- Therefore, in order to achieve the desired six degree angle, the ball should be *curved*.
- If the path of the ball is curved then the ball is experiencing acceleration. What forces are acting on the ball after it is released?
- We turn to physics to better understand the mechanism for curving the ball.

A rolling/sliding ball:

- Suppose a ball (radius R) is rolling and sliding with velocity v_C and angular rotation speed ω_C.



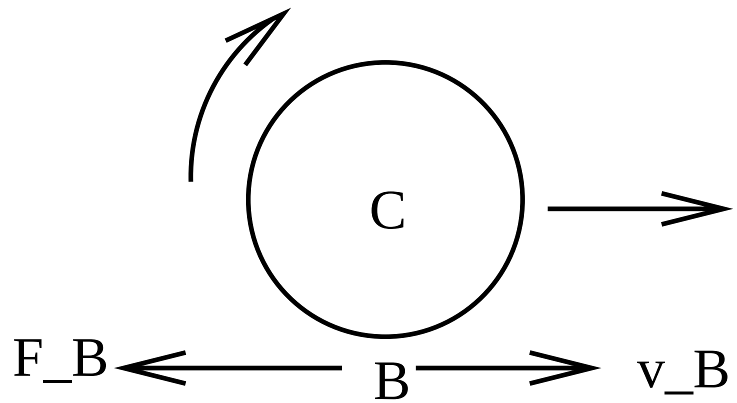
- The velocity of the contact point B is given by:

$$v_B = v_C - \omega_C R$$

- If $v_B = 0$ then the ball is undergoing pure rolling, otherwise there is sliding and frictional forces will act on the ball.

Friction Force:

- A frictional force F_B will act on the ball opposite to the direction of v_B.



- If $v_B > 0$ then ω_C is too small for pure rolling and the friction force takes translational energy and converts it into rotational energy, increasing ω_C and decreasing v_C until $v_B = 0$.
- If $v_B < 0$ then ω_C is too large for pure rolling and the friction force takes rotational energy and converts it into translational energy, increasing v_C and decreasing ω_C until $v_B = 0$.

Energy conservation:

- Energy is conserved when v_C and ω_C have the same sign.
- Energy is destroyed when v_C and ω_C have opposite signs.
- As one expects, it is, therefore, possible for a ball to be sliding forward with a backspin that eventually causes the ball to stop moving. (Easier said than done.)
- There are many bowlers that release the ball with a small amount of backspin. Generally the ball can be seen to reverse its direction of spin as the ball heads down the lane.
- If a small amount of spin is used the ball will generally achieve a pure rolling state before reaching the pins.

Mathematical model: (1 of 4)

- Physical parameters:

$$\begin{aligned} \text{Mass} &= m \\ \text{Ball Radius} &= r \\ \text{Rotational Inertia} &= I = \tfrac{2}{5}mr^2 \\ \text{Coefficient of Friction} &= \mu \\ \text{Gravitational Constant} &= g \end{aligned}$$

- Oil conditions: Although oil patterns vary from one place to the next, typically a bowling lane is oiled from the foul line to a point about fifteen feet in front of the pins.
- This last section has a much higher coefficient of friction due to the lack of oil and often contributes a noticeable redirection of the ball trajectory. (the "extra" spin must last long enough for this to work)

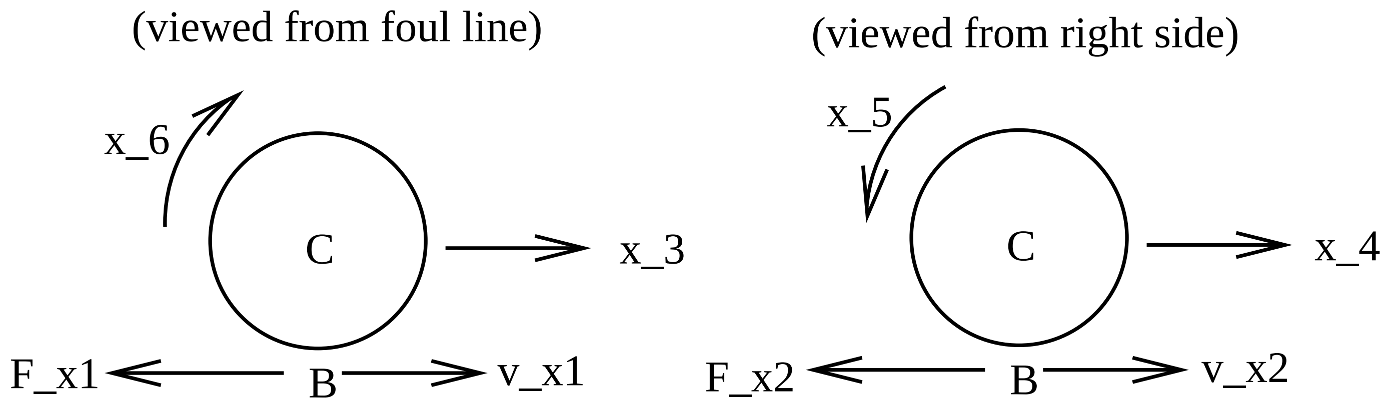
Mathematical model: (2 of 4)

- Variables: We will use a vector $x = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{pmatrix}$

$$
\begin{aligned}
x_1 &= \text{position in lane (from left)} \\
x_2 &= \text{position along lane (from foul line)} \\
x_3 &= \text{velocity in lane (left to right)} \\
x_4 &= \text{velocity along lane (towards pins)} \\
x_5 &= \text{rotation about } x_1 \text{ axis (backspin +)} \\
x_6 &= \text{rotation about } x_2 \text{ axis (rightspin +)}
\end{aligned}
$$

- System of differential equations: One obtains a linear system of first order differential equations by applying Newton's 2nd law to the bowling ball. (Both translational and rotational forms.)

Mathematical model: (3 of 4)



- Friction Forces:

$$F_{x_1} = -\mu \, m \, g \, \mathrm{sgn}(v_{x_1})$$
$$F_{x_2} = -\mu \, m \, g \, \mathrm{sgn}(v_{x_2})$$

- Newton's second law:

$$m \, dx_3/dt = F_{x_1}$$
$$m \, dx_4/dt = F_{x_2}$$

Mathematical model: (4 of 4)

- Rotational form of Newton's second law:

$$I\,dx_6/dt = F_{x_1} r$$

$$I\,dx_5/dt = -F_{x_2} r$$

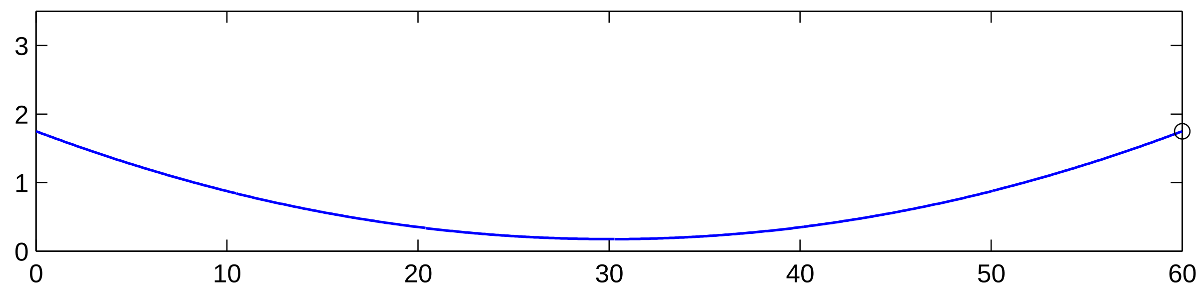
- The complete system of differential equations:

$$\begin{aligned}
\frac{dx_1}{dt} &= x_3 & \frac{dx_2}{dt} &= x_4\\
\frac{dx_3}{dt} &= \frac{F_{x_1}}{m} & \frac{dx_4}{dt} &= \frac{F_{x_2}}{m}\\
\frac{dx_5}{dt} &= \frac{F_{x_2} r}{I} & \frac{dx_6}{dt} &= -\frac{F_{x_1} r}{I}.
\end{aligned}$$

- Notice that the components of acceleration for the ball are piece-wise constant.

Implications of the model:

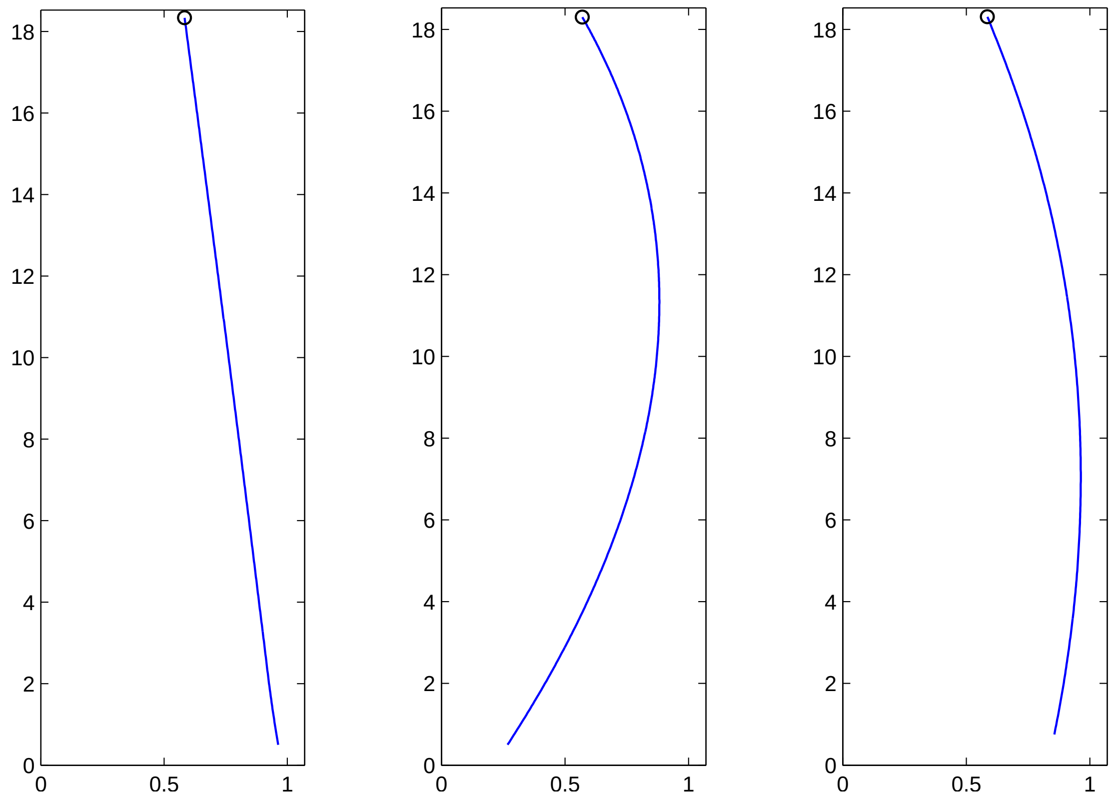
- The ball does not curve faster with more spin, it just curves longer. (takes longer to reach pure rolling)
- The ball follows a parabolic path while it is sliding and a linear path after it achieves pure rolling.
- It is possible to achieve the six degree angle with a pure parabola: ($f(x) = ax^2 + bx + c$, $a = \tan(\pi/30)/60$, $b = -\tan(\pi/30)$, & $c = 1.75$; x in feet)



Parabolic trajectory

Cheating the geometry:

- The model allows us to compare trajectories obtained for various spins, speeds, and release angles. (pictures not to scale)



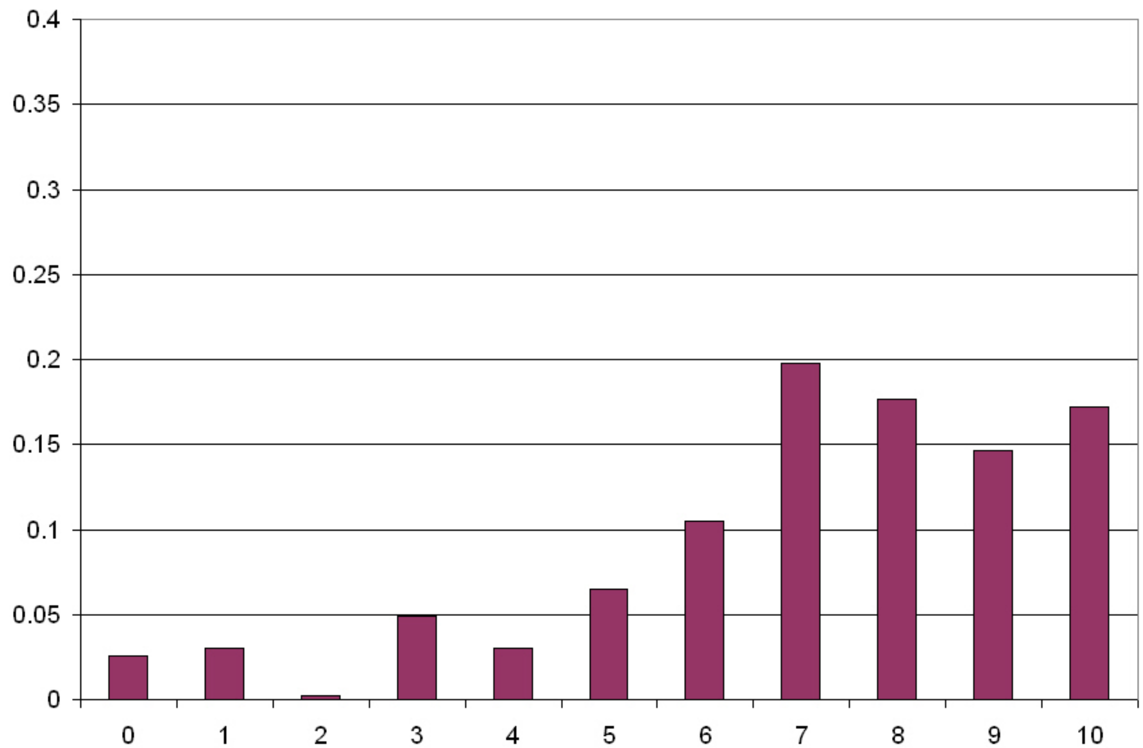
1.60 degrees 5.38 degrees 3.75 degrees

Putting the theory into practice:

- The three trajectories on the previous slide illustrate three stages of my bowling game:
  - Throwing straight and not knowing what I was doing. (Average 120; 17% strikes)
  - Using spin by leaving the thumb out and torquing the ball. (Average 130; 22% strikes)
  - Using spin by getting a personalized ball with fingertip grip. (Average 160; 27% strikes)
- Good bowlers seem to get on the order of 50% strikes. My current method seems to fall short of the six degree angle and is constrained by my level of coordination.

Bowling without spin: (straight throws)

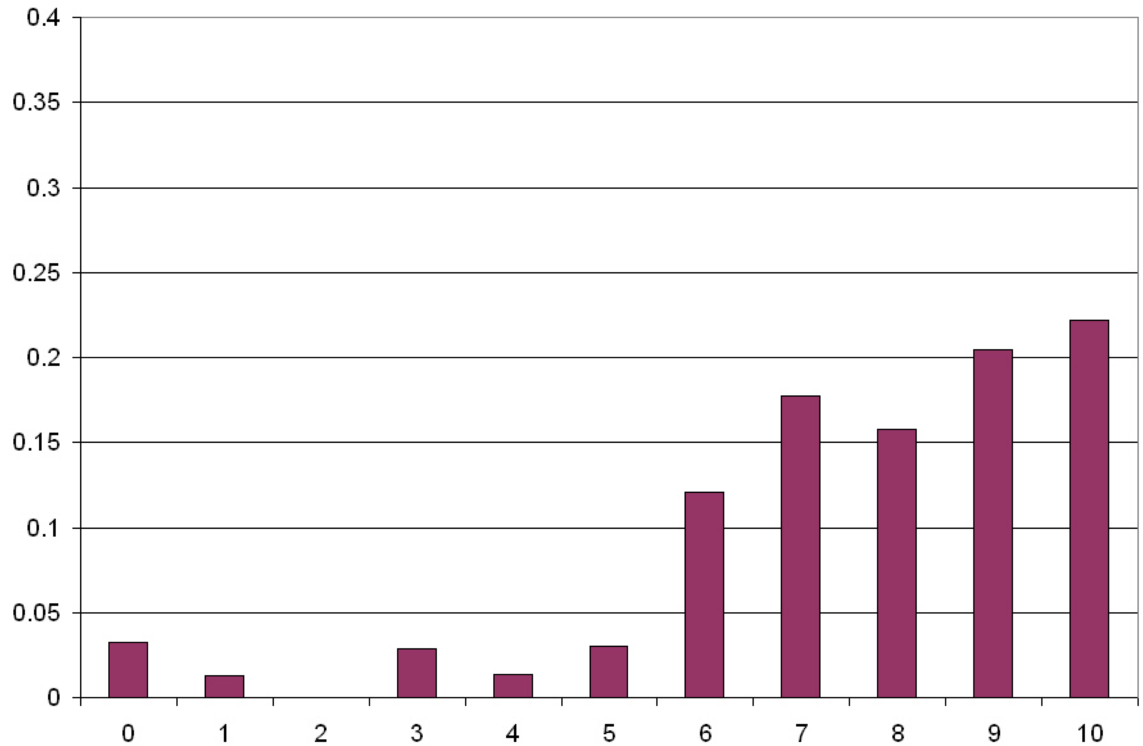
- About 40 games: Average: 120, First-ball average: 7.1 pins.



6/4/2004	Brady Commons	5	10	10	10	8 2	3 6	8 0	8 2	10	5 4	10 7 2
CF: 7, STR: 5			30	58	78	91	100	108	128	147	156	175

Bowling with spin: (but with thumb out)

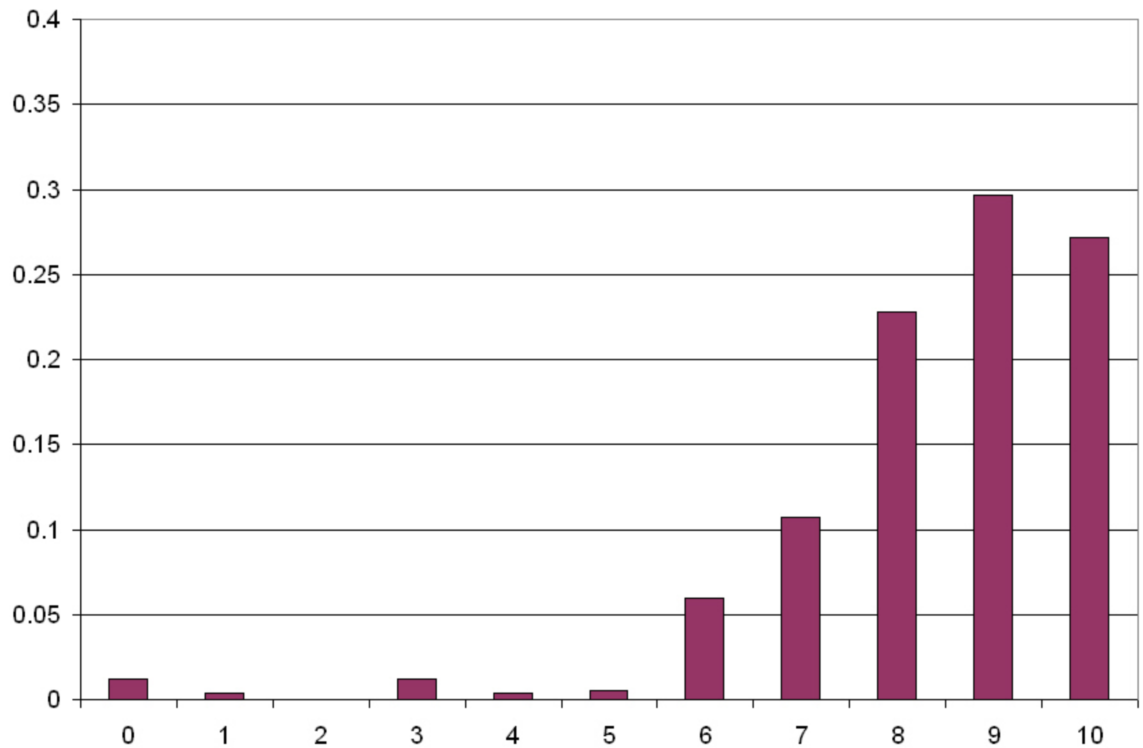
- About 100 games: Average: 130, First-ball average: 7.6 pins.



7/27/2004	Tropicana Lanes	40	7 3	10	7 3	10	10	10	9 1	10	9 1	10 8 0
CF: 10, STR: 6			20	40	60	90	119	139	159	179	199	217

Bowling with spin: (fingertip grip)

- About 60 games: Average: 160, First-ball average: 8.4 pins.



2/15/2005	Tropicana	34	9 | 1	10	10	10	10	10	9 | 1	10	10	9 | 0
CF: 9, STR: 7			20	50	80	110	139	159	179	208	227	236

Conclusion:

- Bowling, like many sports, is enjoyable for players of all skill levels because it is easy to get started and see real results (like bowling a strike), yet most people can play for years without ever coming close to the elusive perfect game.

- The dynamics of the bowling ball, the oil, the gutters, etc. exhibit interesting concepts from math and physics that are accessible to anyone with a background in differential equations.